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An alternative to common envelope evolution

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ABSTRACT

We investigate the evolution of interacting binaries where the donor star is a low–mass giant more massive than its companion. It is usual to assume that such systems undergo common–envelope (CE) evolution, where the orbital energy is used to eject the donor envelope, thus producing a closer binary or a merger. We suggest instead that because mass transfer is super–Eddington even for non–compact companions, a wide range of systems avoid this type of CE phase. The accretion energy released in the rapid mass transfer phase unbinds a significant fraction of the giant's envelope, reducing the tendency to dynamical instability and merging. We show that our physical picture accounts for the success of empirical parametrizations of the outcomes of assumed CE phases.

Key words: binaries: close – stars: evolution

1 INTRODUCTION

Binary systems tend to shrink when they transfer mass from the more massive to the less massive star. The consequent contraction of the donor's Roche lobe leads to high transfer rates, proceeding on a thermal or near–dynamical timescale depending on whether this star is predominantly radiative or convective. There is evidently a tendency for such systems to enter common envelope (CE) evolution, where the donor star engulfs the system (Paczyński 1976; Ostriker 1976; Webbink 1984). The archetypal case arises when the donor star fills its Roche lobe as a giant. The strong dissipation caused by the accretor moving within the common envelope may then produce two effects: (a) the binary orbit shrinks, releasing orbital binding energy, and (b) this energy release may unbind the envelope entirely, stopping the process before the accretor merges with the core of the donor. Features (a) and (b) are highly desirable in explaining the formation of cataclysmic variables (CVs), where a low–mass main sequence star orbits a white dwarf at a separation considerably smaller than the radius of the white dwarf's giant progenitor, and indeed for tight binary formation in general.

Despite this early promise, several decades of strenuous effort have failed to provide convincing evidence that the process works as originally hoped (see Taam & Sandquist 2000, and references therein). Technically the problem is difficult, requiring full 3D hydrodynamics and a careful treatment of the dissipation processes. However there appear to be problems at a more basic level, which are manifest when we consider the most popular parametrization of CE (e.g. Webbink 1984). This compares the envelope binding energy with the change in orbital energy as

$$\frac{GM_{\rm g}M_{\rm e}}{\lambda R_{\rm g}} = \alpha\left(\frac{GM_{\rm c}M_2}{2a_{\rm f}} - \frac{GM_gM_2}{2a_{\rm i}}\right). \quad (1)$$

Here $M_{\rm g}$ is the giant mass, $R_{\rm g}$ its radius, $M_{\rm c}$ its core mass, $M_{\rm e}$ its envelope mass, M_2 is the accretor mass (assumed fixed during CE evolution) and $a_{\rm i},\ a_{\rm f}$ are the initial and final binary separations. Stellar structure calculations specify the dimensionless parameter $\lambda < 1$, which is often taken as a constant, e.g. $\lambda = 0.5$, and $\alpha < 1$ is a dimensionless efficiency parameter. Clearly only the combination $\alpha\lambda$ is important. The problem which emerges is that in many cases the post–CE orbital separation $a_{\rm f}$ is too large, i.e. there was too little orbital energy release to unbind the giant envelope. Formally this problem appears in the requirement $\alpha\lambda > 1$ (values $\alpha\lambda \sim 6$ are not unknown). Nelemans & Tout (2005) show that these problems are still worse when considering the formation of double white dwarf binaries, where it is possible to reconstruct the state of the progenitor binaries. The required values of $\alpha\lambda$ are actually negative in most cases. All these difficulties point to the need for an extra source of energy to unbind the giant envelope.

A clue to this energy source comes from an observed case which CE evolution clearly cannot produce. The low–mass X–ray binary Cyg X–2 consists of a neutron star accreting from a low–mass donor ($\simeq 0.5 - 0.7\,{\rm M_{\odot}}$) in a 9.8 d orbit (see King & Ritter 1999, and references therein). The donor has radius $\simeq 7\,{\rm R_{\odot}}$, and its effective temperature implies luminosity $\simeq 150\,{\rm L_{\odot}}$. Evidently this star was considerably more massive in the recent past and we are now seeing its luminous inner layers before they cool. But the current binary period of 9.8 d makes the binary so wide that far too little orbital energy was available to unbind the envelope, regardless of how wide the pre–CE separation $a_{\rm i}$ was. However in this case the required extra energy source is clear. Assuming that mass transfer began when the donor was the more massive star and still predominantly radiative, mass flowed towards the neutron star on a thermal timescale $\sim 10^6$ yr, giving a transfer rate $\gtrsim 10^{-6}\,{\rm M_{\odot}\,yr^{-1}}$. This is clearly highly super–Eddington. King & Ritter (1999) were able to suggest a plausible evolutionary sequence leading to the current state of Cyg X–2 by assuming that the neutron star accreted only

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at the Eddington rate, the remaining transferred mass being ejected from the system with the specific angular momentum of the neutron star. Podsiadlowski & Rappaport (2000) similarly found a believable evolutionary history for the system starting from a somewhat less evolved donor.

The crucial step in understanding Cyg X–2 is to assume that the accretion energy of some of the transferred matter can eject the remainder, without the system entering CE evolution. Although earlier papers considered the possibility of using accretion energy in this way (cf Han & Webbink 1999) they always concluded that the radiatively driven outflow would be dense and extended enough to act as a surrogate envelope for the accretor, and lead back to CE evolution. However King & Begelman (1999) argued that CE evolution would only occur if a sufficiently dense region of the gas flow on to the accretor overfilled the latter's Roche lobe. They identified the boundary of this dense region as the trapping radius

$$R_{\rm trap} = \frac{\left|\dot{M}_1\right|}{\dot{M}_{\rm edd}} R_2 \,, \tag{2}$$

where the accretion flow first reaches its local Eddington limit ($\dot{M}_{\rm edd}$ is the Eddington rate at the accretor radius R_2, see below) overfilled the accretor's Roche lobe. The trapping radius is effectively identical to the spherization radius defined for disc accretion by Shakura & Sunyaev (1973), who show that disc accretion at super-Eddington rates becomes spherical inside this radius and is a tenuous wind outside it, making it a reasonable estimate for the effective size of any dense envelope (see also section 2).

On this basis King & Begelman (1999) showed that systems where a neutron star or black hole accretes from a more massive star would probably always avoid CE evolution provided that the donor was predominantly radiative, i.e. that mass transfer proceeded on a thermal timescale, as in Cyg X–2. This allows values as high as $10^{-3}\ {\rm M_\odot\,yr^{-1}}$, and SS433 is an example of a system in this state (King, Taam & Begelman 2000; Begelman, King & Pringle 2006). King & Begelman (1999) also suggested that such avoidance of CE evolution might apply even to donors which were largely convective, but did not attempt quantitative estimates.

In this paper we extend the treatment of King & Begelman (1999) to such cases. We note that the mass transfer rates probably exceed the Eddington accretion rate

$$\dot{M}_{\rm edd} = \frac{4\pi R_2 m_{\rm p} c}{\sigma_{\rm T}} \sim 10^{-3} \frac{R_2}{\rm R_\odot} {\rm M_\odot\,yr^{-1}} \tag{3}$$

for gravitational energy release on a main sequence star (here R_2 is the radius of the accretor, $m_{\rm p}$ is the mass of a proton, c the speed of light and $\sigma_{\rm T}$ the Thompson scattering cross-section). Thus it may be possible to avoid CE evolution even in cases where a red giant overflows on to a main sequence star, as required to form cataclysmic variables.

Specifically we investigate here the evolution of systems in which mass transfer occurs from giants on to less massive companions. We find that the trapping radius of the companion (where mass accretion reaches the local Eddington limit) is within its Roche lobe, so CE evolution may be avoided. Our treatment suggests that a comparison of the fractional change of mass with angular momentum offers a more apt parametrization of the change of orbital separation than the energy formalism (equation 1). We note that Nelemans & Tout (2005) have proposed such a formalism (but without an explicit physical reason for the mass loss) which empirically describes the formation of sub–dwarf B star/white–dwarf and white–dwarf/M–dwarf binaries. We shall show that our physical treatment gives similar results, thus offering a possible explanation for its success in this case.

2 EVOLUTIONARY METHOD

We consider donor stars in initial evolutionary states ranging from the start of the Hertzsprung gap to the tip of the asymptotic giant branch.

We define the mass ratio $q = M_1/M_2$ (normally > 1) with M_1 the donor mass and M_2 the accretor mass. The Roche lobe radius ($R_{\rm L}$) as a function of orbital separation (a) for the donor is given by Eggleton (1983)

$$\frac{R_{\rm L}}{a} = \frac{0.49 q^{2/3}}{0.6 q^{2/3} + \ln(1+q^{1/3})} \,. \tag{4}$$

For given masses we can find the radius of the donor when it fills its Roche lobe at a given initial period ($P_{\rm i}$). Using the Eggleton stellar evolution code (Eggleton 1971; Pols et al. 1998, and references therein) we then find the corresponding core mass ($M_{\rm c}$) of the donor at this radius. We assume that mass transfer is so rapid that the core mass does not significantly increase during the mass transfer phase.

The instantaneous mass transfer rate is given by tracking the overfilling of the donor's Roche lobe as given by Paczyński & Sienkiewicz (1972). Substituting their equation (4) into their equation (A26) we find

$$-\dot{M}_1 = \frac{S_1 E(m) W(\mu)}{2P_{\rm orb}} \left(\frac{R_1}{a}\right)^{-\frac{3}{2}} \frac{(M_1+M_2)^{\frac{3}{2}}}{\sqrt{M_1}} \left(\frac{\Delta R_1}{R_1}\right)^3 \tag{5}$$

where S_1 is 0.215; m is the fractional core-mass of the donor ($M_{\rm c}/M_1$); $E(m)$ is a function tabulated by Paczyński & Sienkiewicz (1972); μ is $M_1/(M_1+M_2)$; ΔR_1 is the difference in stellar radius and Roche-lobe radius ($R_{\rm L}$) of the donor and $W(\mu)$ is given by

$$W(\mu) = \frac{\sqrt{\mu}\sqrt{1-\mu}}{\left(\sqrt{\mu}+\sqrt{1+\mu}\right)^4} \left(\frac{\mu}{R_{\rm L}}\right)^3 \,. \tag{6}$$

We let a fraction ψ of the transferred matter be blown away (cf Bhattacharya & van den Heuvel 1991, who call this fraction β). An expression for ψ which accounts for the proportion of matter which must be accreted to power the outflow is given by Han & Webbink (1999) (who call this quantity $1-\beta$)

$$\psi = \begin{cases} 1 - \dfrac{L_{\rm edd}}{|\phi_{\rm R2}|\,\left|\dot{M}_1\right|} - \dfrac{\phi_{\rm L1}}{\phi_{\rm R2}} & \text{if } \left|\dot{\rm M}_1\right| \geq \dot{\rm M}_{\rm edd} \\ 0 & \text{if } \left|\dot{\rm M}_1\right| < \dot{\rm M}_{\rm edd} \end{cases} \,, \tag{7}$$

where $L_{\rm edd}$ is the Eddington luminosity

$$L_{\rm edd} = \frac{4\pi G M_2 m_{\rm p} c}{\sigma_{\rm T}} \,, \tag{8}$$

and $\phi_{\rm L1}$ and $\phi_{\rm R2}$ are the potentials at the inner Lagrangian point and the surface of the accretor respectively and are given by

$$\phi_{\rm L1} = -\frac{GM_1}{a-b} - \frac{GM_2}{b} - \frac{G(M_1+M_2)}{2a^3}(\mu a - b)^2 \,, \tag{9}$$

$$\phi_{\rm R2} = -\frac{GM_1}{a} - \frac{GM_2}{R_2} - \frac{G(M_1+M_2)}{2a^3}\left[\frac{2}{3}R_2^2 + (\mu a)^2\right] \,, \tag{10}$$

where b is the distance from the accretor to the inner Lagrangian point and is given by Warner (1976)

$$\frac{b}{a} = 0.5 - 0.227 \log q \,, \tag{11}$$

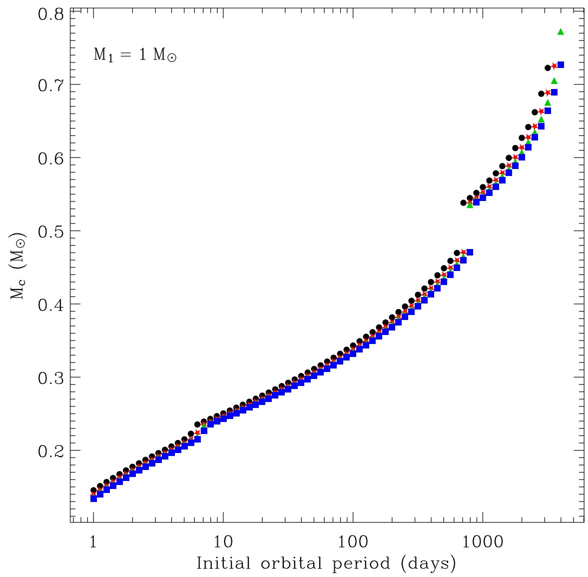

Figure 1. The initial periods at which a $1\,M_\odot$ donor fills its Roche lobe and the corresponding donor core masses for various accretors: $0.2\,M_\odot$ main-sequence stars (black circles), white dwarfs (red stars), $0.6\,M_\odot$ main-sequence stars (green triangles) and $1\,M_\odot$ main-sequence stars (blue squares).

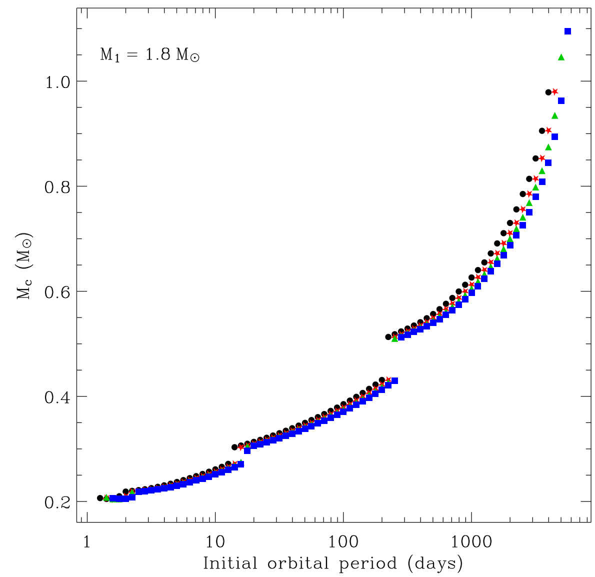

Figure 2. The initial periods at which a $1.8\,M_\odot$ donor fills its Roche lobe and the corresponding donor core masses for various accretors. Symbols are the same as in Fig. 1.

if $0.1 \le q \le 10$ or Kopal (1959) if $q > 10$

$$\frac{b}{a} = w - \frac{w^2}{3} - \frac{w^3}{9}\,, \tag{12}$$

where

$$w^3 = \frac{1}{3(1+q)}\,. \tag{13}$$

If $R_2 \ll a$ equation (7) becomes for $\left|\dot{M}_1\right| \ge \dot{M}_{\rm edd}$

$$\psi = 1 - \frac{\dot{M}_{\rm edd}}{\left|\dot{M}_1\right|}\,. \tag{14}$$

This is equivalent to assuming that the transferred matter cannot be accreted at a rate faster than the Eddington rate (equation 3), and that all transferred matter is accreted for sub-Eddington transfer rates. We consider below how much angular momentum this mass carries off.

The trapping radius of the accretor, equation (2), depends only on the mass transfer rate from the donor. During the evolution we compare the trapping radius with the Roche radius of the accretor found using equation (4) with the mass ratio now inverted. If the trapping radius is greater than the Roche lobe we assume that the system enters CE evolution and do not follow its evolution. In the rest of the paper we show only systems where the trapping radius never overfills the Roche lobe.

The resultant change in orbital separation depends on the specific angular momentum of the lost mass. If the transferred matter forms a disc and mass is lost with circular symmetry from this disc, the appropriate value is the specific angular momentum of the accretor. As the accretor is further from the centre of mass this process shrinks the binary. However if the transferred matter cannot circularize and form a disc, i.e. its circularization radius

$$R_{\rm circ} \simeq a(1+q)\left(\frac{b}{a}\right)^4 \tag{15}$$

(where b is the distance of the accretor from the inner Lagrange point L_1, cf Frank, King & Raine 2002) is smaller than the trapping radius $R_{\rm trap}$, the lost matter has specific angular momentum closer to that of L_1. This follows because, unless a disc forms, the gas stream from the donor effectively conserves its specific angular momentum at a value close to that of L_1 (King, Whitehurst & Frank 1990). The donor thus loses matter with specific angular momentum slightly less than its centre of mass, i.e. the donor slightly increases its specific angular momentum. Since the binary mass also drops, this tends to lead to expansion, as equation (20) of van Teeseling & King (1998) shows. This of course then makes $R_{\rm circ}$ increase above $R_{\rm trap}$, creating a stable feedback so that the binary subsequently evolves with $R_{\rm circ} \simeq R_{\rm trap}$. This occurs particularly in systems with large mass ratios, where most of the transferred matter is lost. These systems thus do not merge unless the trapping radius exceeds the Roche radius of the accretor and a CE phase occurs.

To demonstrate the trapping radius is the boundary of the dense region we compute the density $\rho(R)$ in the outflow from the trapping radius. This is essentially a spherical wind with constant velocity equal to the escape velocity $v_{\rm esc}(R_{\rm trap} = (2GM/R_{\rm trap})^{1/2}$ from the trapping radius. Thus

$$\rho(R) = \frac{\dot{M}_{\rm out}}{4\pi R^2 v_{\rm esc}(R_{\rm trap})}. \tag{16}$$

Now using (2) gives

$$\rho = \frac{\dot{M}_{\rm edd}}{4\pi (GM_2)^{1/2} R_2^{3/2}}\left(\frac{\dot{M}_{\rm edd}}{\dot{M}_{\rm out}}\right)^{1/2}\left(\frac{R_{\rm trap}}{R}\right)^2. \tag{17}$$

Here we use (3) to replace $\dot{M}_{\rm edd}$ in the first factor on the rhs, giving

$$\rho = \frac{c m_p}{\sigma_{\rm T}(GM_2R_2)^{1/2}}\left(\frac{\dot{M}_{\rm edd}}{\dot{M}_{\rm out}}\right)^{1/2}\left(\frac{R_{\rm trap}}{R}\right)^2 \tag{18}$$

or

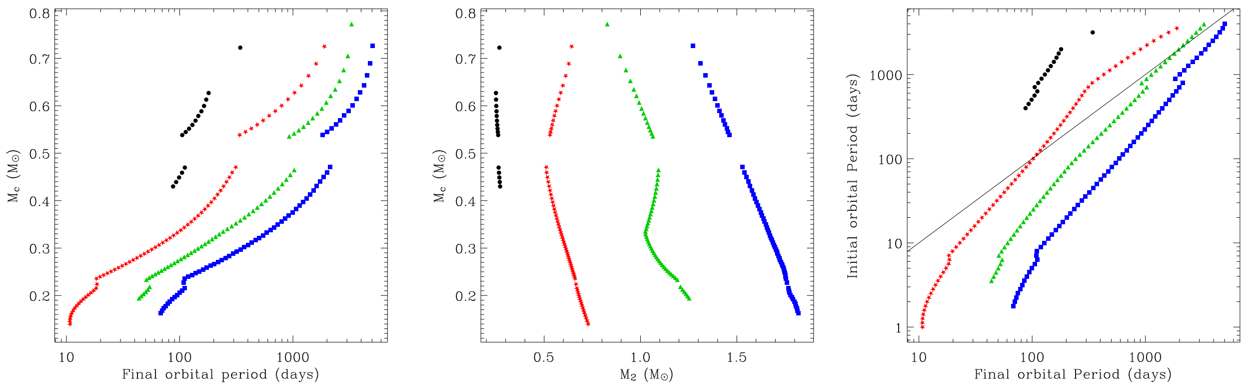

Figure 3. Final period and accretor mass versus core mass, and final versus initial period, for mass-transfer from a 1 $M_\odot$ donor. Symbols as in Fig. 1. Diagonal lines indicate where the final and initial orbital periods are equal.

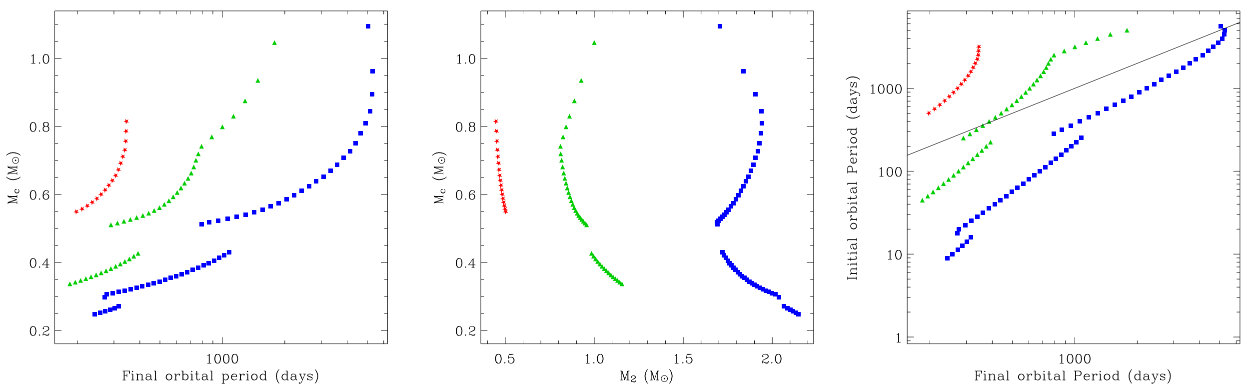

Figure 4. Final period and accretor mass versus core mass, and final versus initial period, for mass-transfer timescale from a 1.8 $M_\odot$ donor. Symbols as in Fig. 1. Diagonal lines indicate where the final and initial orbital periods are equal.

$$\rho = \frac{3.8\times 10^{-8}}{(m_2 r_2)^{1/2}}\left(\frac{\dot M_{\rm edd}}{\dot M_{\rm out}}\right)^{1/2}\left(\frac{R_{\rm trap}}{R}\right)^2 \ {\rm g\ cm^{-3}} \qquad (19)$$

where m_2, r_2 are M_2, R_2 in solar units. (Note that eqn 18 reproduces the familiar result that the optical depth through an outflow at the Eddington rate is of order unity at the Schwarzschild radius, i.e. setting $\dot M_{\rm out} = \dot M_{\rm edd}, R_2 = R_{\rm trap} = R_s = 2GM/c^2$ gives $\tau = \rho\sigma_{\rm T} R_s/m_p \sim 1$.)

The density (19) implies a drag force $\sim \pi R_2^2 \rho v^2$, where R_2, v are the companion's radius and orbital velocity, or equivalently a drag torque

$$-\dot J \sim \pi R_2^2 a \rho v^2 \qquad (20)$$

where a is the separation. The companion's angular momentum is

$$J \sim (4/3)\pi\rho_2 f^3 a^3 (a^2/P) \qquad (21)$$

where ρ_2 is its mean density (writing its radius as fa). Using this in $\dot J$ we get the drag timescale as

$$t_{\rm drag} \sim -\frac{J}{\dot J} \sim \frac{\rho_2}{\rho} f P \qquad (22)$$

In other words, the drag is slow compared with the dynamical timescale P provided that $\rho \ll \rho_2$. Since the companion is (at worst) a MS star with $\rho_2 \sim 1$ we see from (19) that the drag is small provided $a \gg R_{\rm trap}$ since $\dot M_{\rm out} \gg \dot M_{\rm edd}$ by hypothesis.

Once the donor fills its Roche lobe we consider the transfer of small portions of the envelope ΔM. The change in orbital separation of the system Δa is

$$\frac{\Delta a}{a} = 2\frac{\Delta j}{j} + \frac{\Delta(M_1+M_2)}{M_1+M_2} - 2\frac{\Delta M_1}{M_1} - 2\frac{\Delta M_2}{M_2}\,, \qquad (23)$$

where

$$\frac{\Delta j}{j} = -\frac{\psi q \Delta M \gamma}{(M_1+M_2)}\,, \qquad (24)$$

and

$$\gamma = \begin{cases} 1 & \text{if } j = j_{\rm acc} \\ \left[1 - \frac{b}{a}\left(\frac{1+q}{q}\right)\right]^2 & \text{if } j = j_{\rm L1} \end{cases}\,, \qquad (25)$$

and equation (23) becomes

$$\frac{\Delta a}{a} = -\frac{2\psi q\Delta M\gamma}{(M_1+M_2)} - \frac{\psi\Delta M}{M_1+M_2} + \frac{2\Delta M}{M_1} - \frac{2(1-\psi)\Delta M}{M_2}\,. \qquad (26)$$

We choose ΔM so that Δa is always less than 1 per cent of a. This formulation gives the same results as the calculations of Bhattacharya & van den Heuvel (1991) and Kalogera & Webbink (1996) in which the final orbital separation ($a_{\rm f}$) (after transfer of mass with a part ψ lost with the angular momentum of the accretor) is given by

$$\frac{a_{\rm f}}{a_{\rm i}} = \left[\frac{M_{2\rm i}}{M_{2\rm i} + (1-\psi)(M_{1\rm i} - M_{1\rm f})}\right]^{\left(\frac{2}{1-\psi}\right)} \times$$

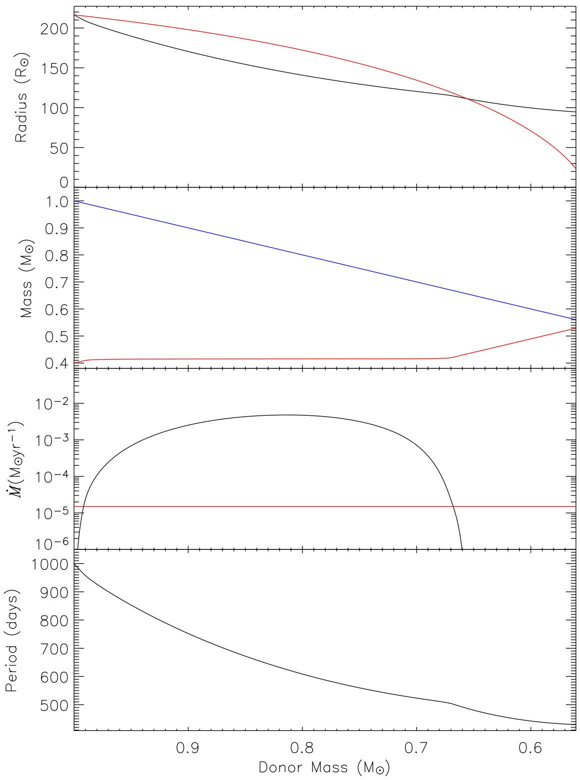

Figure 5. Figure showing the evolution of a $1\,\mathrm{M}_\odot$ donor and a $0.4\,\mathrm{M}_\odot$ white dwarf with an initial orbit period of 1000 days. The top panel shows the evolution of the radius of the donor (red line) and the donor's Roche-lobe (black) with donor mass. The second panel shows the masses of the components (blue and red for donor and accretor respectively). The third panel shows the mass transfer rate and the Eddington limit (red line). The bottom panel shows the evolution of the orbital period.

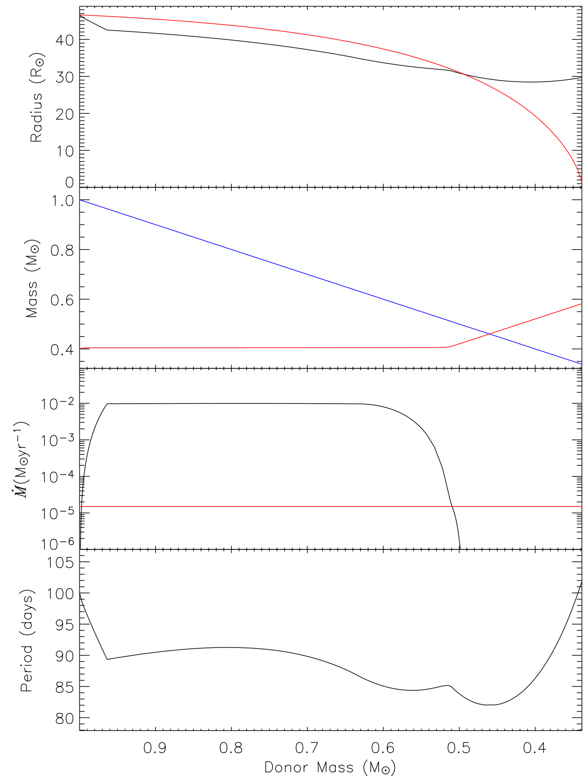

Figure 6. Figure showing the evolution of a $1\,\mathrm{M}_\odot$ donor and a $0.4\,\mathrm{M}_\odot$ white dwarf with an initial orbit period of 100 days. The nomenclature is the same as Figure 5.

$$\left[\frac{M_{1\mathrm{i}} + M_{2\mathrm{i}}}{M_{1\mathrm{i}} + M_{2\mathrm{i}} - \psi(M_{1\mathrm{i}} - M_{1\mathrm{f}})}\right] \left(\frac{M_{1\mathrm{i}}}{M_{1\mathrm{f}}}\right)^2, \quad (27)$$

or by

$$\frac{a_\mathrm{f}}{a_\mathrm{i}} = \frac{M_{1\mathrm{i}} + M_{2\mathrm{i}}}{M_{1\mathrm{f}} + M_{2\mathrm{i}}} \left(\frac{M_{1\mathrm{i}}}{M_{1\mathrm{f}}}\right)^2 \exp\left[\frac{-2(M_{1\mathrm{i}} - M_{1\mathrm{f}})}{M_{2\mathrm{i}}}\right], \quad (28)$$

if ψ is one. The subscript i indicates the initial value before mass transfer and $M_{1\mathrm{f}}$ is the final donor mass assumed to be donor core mass upon filling its Roche lobe. The equations above are identical with equations (A.5) and (A.7) of Bhattacharya & van den Heuvel (1991) with the substitutions $\psi \rightarrow \beta$ and appropriate notation for the initial and final masses (e.g. $M_{1i} \rightarrow m_1^0$).

After each change in orbital separation we calculate the new donor radius, the corresponding new mass transfer rate (equation 5) and γ. The donor radius is found by using the radius-mass exponents of the donor. Hjellming & Webbink (1987) calculated radius-mass exponents ζ for polytropic donor envelopes losing mass rapidly. Soberman, Phinney & van den Heuvel (1997) showed that these can be replaced by functions of fractional core masses (m). We use the following prescription which is accurate to better than a percent

$$\zeta = \frac{2}{3}\left(\frac{m}{1-m}\right) - \frac{1}{3}\left(\frac{1-m}{1+2m}\right) - 0.03m$$

$$+ 0.2\left[\frac{m}{1+(1-m)^{-6}}\right]. \quad (29)$$

While condensed polytropic models (Hjellming 1989, 1990) generally represent the response of red giants well, there are some cases where numerical models predict expansion. The resulting larger transfer rate and trapping radius would increase the chance of entering CE evolution. Evidently one would need to check for this in individual cases.

3 RESULTS

We consider the evolution of $1\,\mathrm{M}_\odot$ and $1.8\,\mathrm{M}_\odot$ donors transferring matter to four possible accretors. We choose these donors because they have degenerate cores on the giant branch. The accretors are (a) a low-mass main-sequence star with an initial mass of $0.2\,\mathrm{M}_\odot$ and a radius of $0.2\,\mathrm{R}_\odot$, (b) a white dwarf of mass $0.4\,\mathrm{M}_\odot$ and a radius of 10000 km, (c) a main-sequence star of $0.6\,\mathrm{M}_\odot$ with an a radius of $0.6\,\mathrm{R}_\odot$, and finally a solar–type star with initial mass $1\,\mathrm{M}_\odot$ and radius $1\,\mathrm{R}_\odot$.

Figures 1 and 2 show the periods at which the donors fill their Roche lobes and their corresponding core masses. Black circles indicate the low-mass main-sequence companion, red stars the white dwarf, green triangles the $0.6\,\mathrm{M}_\odot$ main-sequence star and blue squares the Sun-like star.

Figure 3 shows the outcomes when mass is lost from a $1\,\mathrm{M}_\odot$ donor. Most low-mass main-sequence stars end up with the trapping radius greater than their Roche-lobe radii and so enter CE evolution, producing a merged object with a portion of the donor

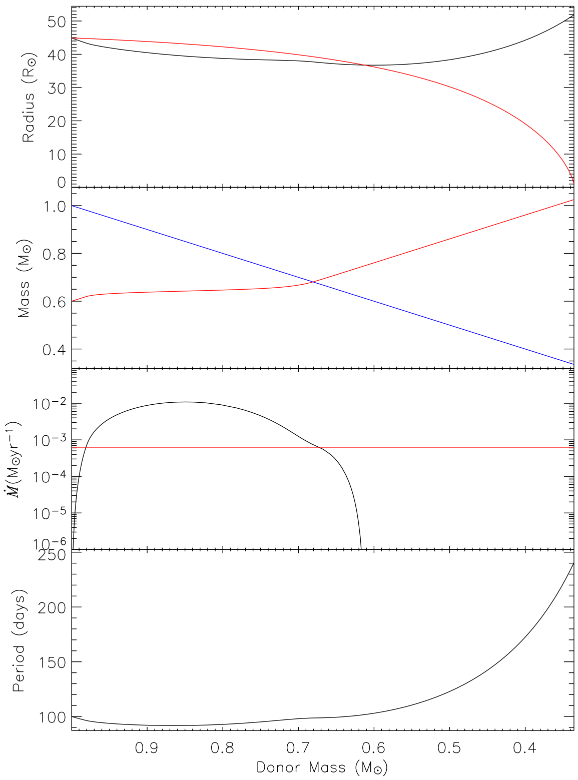

Figure 7. Figure showing the evolution of a 1 $M_{\odot}$ donor and a 0.6 $M_{\odot}$ accretor with an initial orbit period of 100 days. The nomenclature is the same as Figure 5.

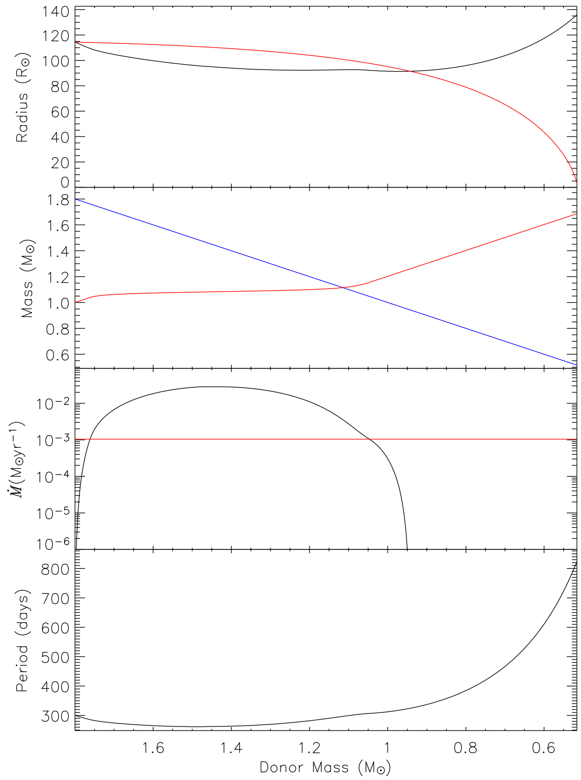

Figure 8. Figure showing the evolution of a 1.8 $M_{\odot}$ donor and a 1.0 $M_{\odot}$ accretor with an initial orbit period of 300 days. The nomenclature is the same as Figure 5.

envelope ejected from the system before the CE phase. For the white dwarf companions around 0.2 $M_{\odot}$ is accreted during the entire evolution and the majority of this after the dynamical mass-transfer phase (see below). The more massive main-sequence stars accrete significant mass. This mass is accreted, however, after the dynamical mass-transfer such that the stars can cool the material sufficiently as they accrete it. In a number of these evolutions the mass-ratio is inverted during the dynamical mass-transfer phase which brings an abrupt end to the dynamical phase. The systems then evolve to longer periods (see below).

Figure 4 shows the outcomes when mass is lost from a 1.8 $M_{\odot}$ donor. Only some of the white-dwarf and main-sequence companions survive. The white-dwarf accretors gain little mass while the main-sequence accretors gain significantly at the end of the dynamical mass-transfer phase (see below). We now consider the evolution of specific systems which illustrate the general trends shown in these sequences.

Figure 5 shows the evolution of a white-dwarf accretor with an initial orbital period of 1000 days. The donor overfills its Roche lobe on the asymptotic giant branch (AGB) with a core mass of 0.56 $M_{\odot}$. After transferring a small portion of mass on the nuclear expansion timescale for the giant the donor becomes sufficiently oversized with respect to its Roche lobe that dynamical mass transfer occurs. During this phase 0.3 $M_{\odot}$ of the envelope is transferred reaching peak mass transfer rates of just under $10^{-2}\,M_{\odot}\mathrm{yr}^{-1}$. The white dwarf accretes at the Eddington rate during this phase and as it is a rapid phase (a timescale of order 100 yrs) the white dwarf does not gain much mass compared to its initial mass. After this dynamical phase however, the star has contracted back inside its Roche lobe and mass transfer is limited to the nuclear expansion of the envelope again. We consider this portion of the evolution to be conservative (all transferred mass is accreted) and the remaining 0.1 $M_{\odot}$ of the envelope is accreted on to the white-dwarf.

Figure 6 shows the evolution of a white-dwarf accretor with an initial orbital period of 100 days. The donor overfills its Roche lobe on the giant branch with a core mass of 0.35 $M_{\odot}$. During the dynamical mass-transfer phase 0.45 $M_{\odot}$ of the envelope is transferred reaching peak mass transfer rates of $10^{-2}\,M_{\odot}\mathrm{yr}^{-1}$. At this mass-transfer rate the trapping radius (which only depends on the transfer rate) fills the circularization radius of the accretor. The system then switches to losing mass with the specific angular momentum of the inner Lagrangian point which tends to widen the binary (see bottom panel of Figure 6). The evolution through this phase then continues so that $R_{\rm trap}$ is the same as $R_{\rm circ}$. If the trapping radius is larger than the circularization radius then the binary widens and the transfer rate drops while if is smaller the binary tightens and the transfer rate increases. After this dynamical phase the remaining 0.18 $M_{\odot}$ of the envelope is accreted on to the white-dwarf. As the mass ratio is inverted the binary becomes wider.

Once the dynamical mass-transfer phase has finished the evolution of the system will proceed much more slowly. Clearly the assumption that the core mass stays constant is no longer valid. The only effect this will have on the calculations, however, is that less mass is available to transfer. This will result in evolution ending sooner and the systems not widening as much after the dynamical mass-transfer stage. In the case of white-dwarf accretors the mass transfer has been assumed to be conservative. During the dynamical mass-transfer phase the accretion rates will be much higher than those required for stable nuclear burning (see Kahabka & van den Heuvel 1997) but afterwards may be low enough for nova behaviour. In this case the final part of the mass-transfer may be non-conservative and the systems would end up tighter i.e. shorter periods.

Figure 7 shows the evolution of a $0.6\,\mathrm{M}_\odot$ main-sequence star with an initial orbital period of 100 days. The donor overfills its Roche lobe on the giant branch with a core mass of $0.34\,\mathrm{M}_\odot$. During the dynamical mass-transfer phase $0.3\,\mathrm{M}_\odot$ of the envelope is transferred reaching peak mass transfer rates of under $10^{-2}\,\mathrm{M}_\odot\mathrm{yr}^{-1}$. Near the end of this dynamical phase however, the mass ratio has been inverted and mass transfer widens the binary rapidly halting the dynamical mass-transfer phase. The remaining $0.35\,\mathrm{M}_\odot$ of the envelope is then accreted on to the main-sequence accretor raising its mass to over $1\,\mathrm{M}_\odot$. The evolution of this system is substantially different to that of standard CE evolution where the accretor would be assumed not to gain in mass.

Figure 8 shows the evolution of a binary composed of a 1.8 and a $1.0\,\mathrm{M}_\odot$ main-sequence star with an initial orbital period of 300 days. As in Figure 7, near the end of the dynamical phase the mass ratio is inverted. The system then evolves to longer periods.

The radial response of low-mass main-sequence star to accretion is significantly different to that of a high-mass star. As Fujimoto & Iben (1989) show a low-mass star with a convective envelope does not significantly expand on accretion and could even contract. In our scenario the entropy of the transferred mass will be higher than the main-sequence surface but it will be subject to a far higher surface gravity. Regardless of the detailed radial response of the secondary it is clear from Figures 7 and 8 that only a relatively small portion of mass is accreted in the build-up to and during dynamical mass-transfer and so we do not expect significantly expand during this process.

3.1 Parametrizations

Treatments of CE evolution have generally been expressed in terms of empirical parametrizations (Webbink 1984; Nelemans & Tout 2005). Here we compare our results with these parametrizations. This serves two purposes. First, where the parametrizations are successful, our treatment should agree, and indeed offer an explanation for the success. Second, a simple and robust parametrization would give a succinct way of summarizing complex physics. As these parametrizations assume no accretion onto the secondary we make the same assumption in order to compare our model to their parametrizations.

Our picture predicts a total change in angular momentum (ΔJ) proportional to the total change in system mass (ΔM)

$$\frac{\Delta J}{J} = \gamma \frac{\Delta M}{M} \,. \tag{30}$$

This is the form assumed empirically by Nelemans & Tout (2005). Our physical picture of the process allows us to specify an upper limit on γ which we can compare with the results they get by applying their parametrization to observed systems.

Assuming that the entire envelope of the donor is lost (30) becomes

$$\frac{\Delta J}{J} = \gamma \frac{M_\mathrm{env}}{M_\mathrm{c}+M_2} \,. \tag{31}$$

The orbital angular momentum of the system is

$$J = M_1 M_2 \sqrt{\frac{Ga}{M_1+M_2}} \,, \tag{32}$$

so the left hand side of equation (30) is

$$\frac{\Delta J}{J_\mathrm{i}} = 1 - \frac{M_\mathrm{1f}}{M_\mathrm{1i}} \frac{M_\mathrm{2f}}{M_\mathrm{2i}} \sqrt{\frac{(M_\mathrm{1i}+M_\mathrm{2i})a_\mathrm{f}}{(M_\mathrm{1f}+M_\mathrm{2f})a_\mathrm{i}}} \,. \tag{33}$$

If the matter is lost with the angular momentum of the accretor[1] then taking M_2 as fixed, and using equation (28) this becomes

$$\frac{\Delta J}{J_\mathrm{i}} = 1 - \frac{M_\mathrm{1i}+M_2}{M_\mathrm{1f}+M_2} \exp\left(-\frac{M_\mathrm{env}}{M_2}\right) . \tag{34}$$

Hence in this case we have

$$\gamma = \frac{M_\mathrm{1i}+M_2}{M_\mathrm{env}} - \frac{(M_\mathrm{1i}+M_2)^2}{M_\mathrm{env}(M_\mathrm{c}+M_2)} \exp\left(-\frac{M_\mathrm{env}}{M_2}\right) . \tag{35}$$

This is an upper limit as we assume that the lost matter carries the specific angular momentum of the accretor which will have a higher specific angular momentum than the inner Lagrangian point. Hence if there is any mass loss without a disc the specific angular momentum of the lost matter will be smaller and the total ΔJ will be less.

Fig. 9 shows γ versus donor mass for various core and accretor masses. Nelemans & Tout (2005) show that observed close white–dwarf binaries are well described by the choice $\gamma \simeq 1.5-1.75$. Since many of the values in Fig. 9 are within this range we conclude that the evolution of close white-dwarf binaries is compatible with the idea that all the transferred matter is ejected with the specific angular momentum of the accretor.

Nelemans & Tout (2005) considered neutron–star progenitors in the antecedents of low–mass X–ray binaries and argued that γ values slightly above 1.5 allow formation. From equation (35) we find that a typical system with $M_\mathrm{env} = 6\,\mathrm{M}_\odot$, $M_\mathrm{c} = 2.5\,\mathrm{M}_\odot$, $M_2 = 1\,\mathrm{M}_\odot$ has $\gamma = 1.56$. This is low enough not to merge, but high enough to form a tight binary. Again these systems are compatible with all the transferred matter being ejected with the angular momentum of the accretor.

We have not considered accretion on to neutron stars but such accretion is likely to be super–Eddington so we would expect most of the mass to be ejected with the specific angular momentum of the neutron star. This evolution is similar to Cyg X–2. Applying the results of King & Ritter (1999) with $M_{1i} = 3.6\,\mathrm{M}_\odot,\ M_\mathrm{env} = 3\,\mathrm{M}_\odot,\ M_2 = 1.4\,\mathrm{M}_\odot$, equation (35) gives a γ of 1.2.

We now turn to the comparison with the energy or $\alpha\lambda$ formalism (Webbink 1984). Making the same assumptions as for equation (35) (all mass ejected with specific angular momentum of the accretor) we find

$$\frac{1}{\alpha\lambda} = \left\{ \frac{M_\mathrm{c}+M_2}{M_\mathrm{1i}+M_2} \left(\frac{M_\mathrm{c}}{M_\mathrm{1i}}\right)^2 \exp\left[\frac{2\,(M_\mathrm{1i}-M_\mathrm{c})}{M_2}\right] - \frac{M_\mathrm{1i}}{M_\mathrm{c}} \right\} \frac{M_\mathrm{c} M_2\,(R_\mathrm{1i}/a_\mathrm{i})}{2M_\mathrm{1i}\,(M_\mathrm{1i}-M_\mathrm{c})} \,, \tag{36}$$

where $R_\mathrm{1i}/a_\mathrm{i}$ is given by equation (4) and is a function of only the initial mass ratio (M_1i/M_2). Fig. 10 shows the values of $(\alpha\lambda)^{-1}$ using these assumptions. The solid lines indicate the likely range of $\alpha\lambda$ of $0.5-1.0$. Again one sees that at least some choices of $\alpha\lambda$ produce the same outcomes as predicted by our calculations. However, it is clear that for different accretor masses only a small range of donor masses are possible. This contrasts to the broad range of donor masses possible with the γ prescription. Clearly γ is not a constant but a free parameter which weakly constrains the possible

[1] van der Sluys, Verbunt & Pols (2006) have applied the method of Nelemans & Tout (2005) to a larger number of systems and find the observed double white-dwarfs can be well explained by assuming that envelope ejection occurs with the specific angular momentum of the accretor.

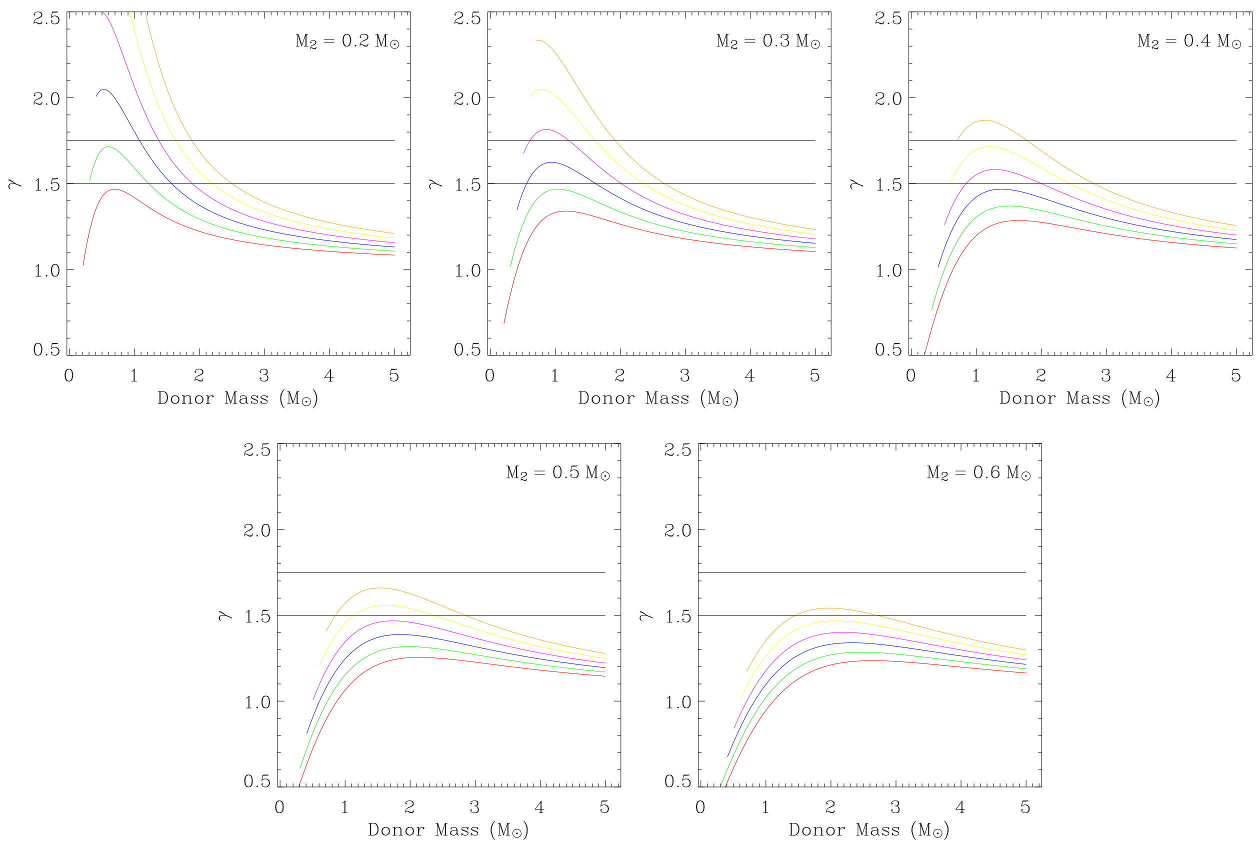

Figure 9. Values of $\gamma_{\rm max}$ for accretors ranging from 0.2 to 0.6 $M_{\odot}$. All matter lost from the system is assumed to carry the angular momentum of the accretor. Line colours indicate donor core masses of 0.2 (bottom) to 0.7 $M_{\odot}$ (top) in increments of 0.1 $M_{\odot}$. Solid horizontal lines show the range for γ suggested by Nelemans & Tout (2005).

progenitors of a system compared to the $\alpha\lambda$ formalism. Our calculations may offer a physical justification for these parametrizations. However it is clear that neither parametrization gives a full description of our results, which is perhaps not surprising.

4 CONCLUSIONS

We have studied binary evolution in cases which are normally thought to lead to common–envelope evolution. We have shown instead that the accretion luminosity is in many cases able to expel much of the super–Eddington mass transfer without catastrophic orbital shrinkage. Treatments of CE evolution have hitherto relied on empirical parametrizations (e.g. Webbink 1984; Nelemans & Tout 2005). In cases where these are successful in describing observed systems our theoretical treatment agrees quite well with the adopted parametrizations. This suggests that our ideas give a reasonable description of the prior evolution of cataclysmic variables, double white dwarf binaries and low–mass X–ray binaries.

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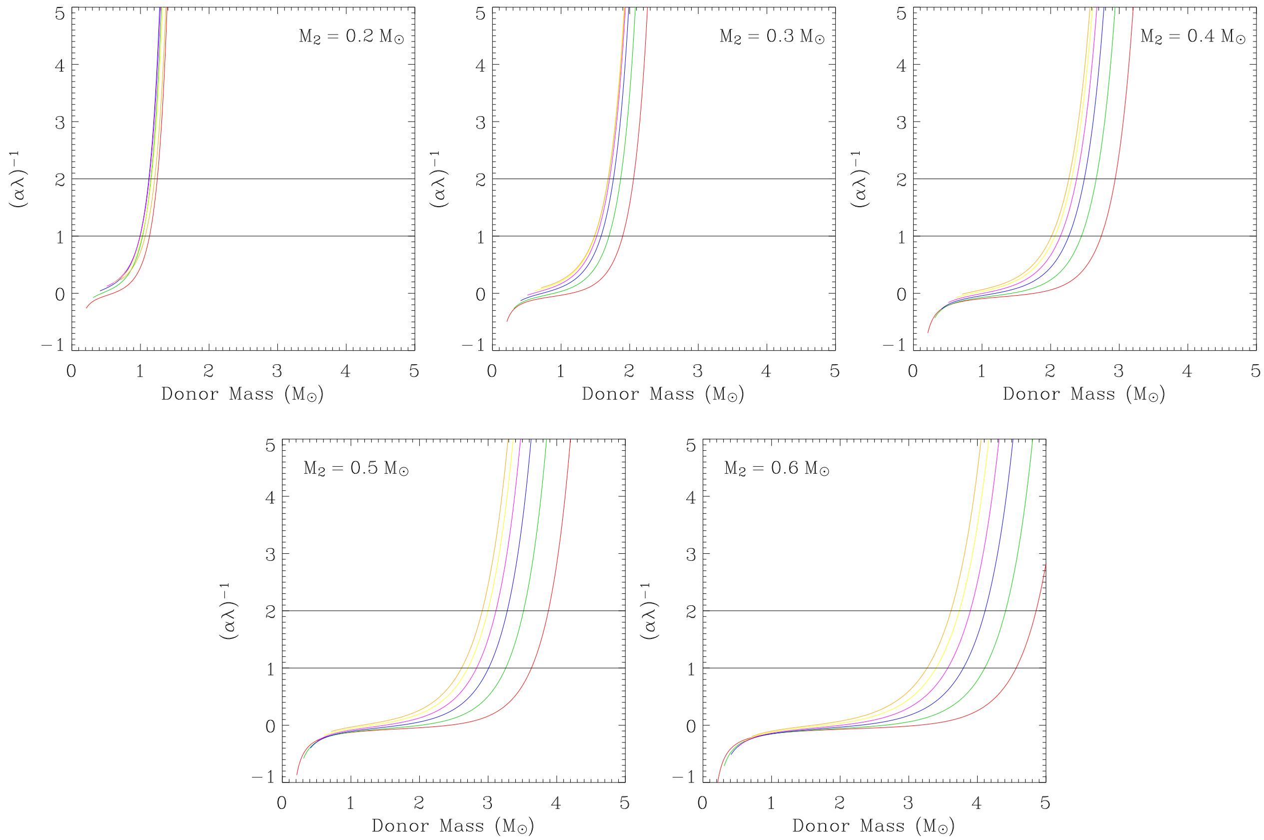

Figure 10. Values of $(\alpha\lambda)^{-1}$ for accretors ranging from 0.2 to 0.6 $M_\odot$. All the transferred matter is assumed lost from the system carrying the specific angular momentum of the accretor. Line colours indicate donor core masses of 0.2 (bottom) to 0.7 $M_\odot$ (top) in increments of 0.1 $M_\odot$. Solid horizontal lines show the typical values assumed for $\alpha\lambda$ of 0.5–1.0.

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